SF1683 HT25 DIFFERENTIALEKVATIONER OCH TRANSFORMMETODER

FORMELBLAD

Trigonometriska formler

(1) $\sin(\pi - \alpha) = \sin\alpha$

(2) $\cos(\pi - \alpha) = -\cos\alpha$

(3) $\sin n\pi = 0, \quad n \in \mathbb{Z}$

(4) $\cos n\pi = (-1)^n, \quad n \in \mathbb{Z}$

(5) $\sin\alpha = \cos\left(\frac{\pi}{2} - \alpha\right)$

(6) $\tan\alpha = \cot\left(\frac{\pi}{2} - \alpha\right)$

(7) $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$

(8) $\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$

(9) $\cos^2\alpha + \sin^2\alpha = 1$

(10) $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

(11) $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

(12) $\sin 2\alpha = 2\sin\alpha\cos\alpha$

(13) $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

(14) $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

(15) $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

(16) $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha + \beta) + \cos(\alpha - \beta)]$

(17) $\sin^2\alpha = \frac{1}{2} - \frac{1}{2}\cos 2\alpha$

(18) $\cos^2\alpha = \frac{1}{2} + \frac{1}{2}\cos 2\alpha$

(19) $\sin^3\alpha = \frac{1}{4}(3\sin\alpha - \sin 3\alpha)$

(20) $\cos^3\alpha = \frac{1}{4}(3\cos\alpha + \cos 3\alpha)$

(21) $e^{i\alpha} = \cos\alpha + i\sin\alpha$

(22) $\cos\alpha = \frac{1}{2}\left(e^{i\alpha} + e^{-i\alpha}\right)$

(23) $\sin\alpha = \frac{1}{2i}\left(e^{i\alpha} - e^{-i\alpha}\right)$

(24) $\cosh\alpha = \frac{1}{2}\left(e^{\alpha} + e^{-\alpha}\right)$

(25) $\sinh\alpha = \frac{1}{2}\left(e^{\alpha} - e^{-\alpha}\right)$

(26) $\cosh^2\alpha - \sinh^2\alpha = 1$

Integraler

(27) $\int x^p\,dx = \frac{1}{p+1}x^{p+1}, \quad p \neq -1$

(28) $\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b|$

(29) $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$

(30) $\int \frac{1}{ax^2+bx+c}\,dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}}$

(31) $\int \frac{1}{(x+a)(x+b)}\,dx = \frac{1}{b-a}\ln\frac{a+x}{b+x}, \quad a \neq b$

(32) $\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2\ln\left|x + \sqrt{x^2 \pm a^2}\right|$

(33) $\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2 - x^2}}$

(34) $\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$

(35) $\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}$

(36) $\int x^p \ln ax\,dx = x^{p+1}\left(\frac{\ln ax}{p+1} - \frac{1}{(p+1)^2}\right), \quad p \neq -1$

(37) $\int \frac{\ln ax}{x}\,dx = \frac{1}{2}(\ln ax)^2$

(38) $\int e^{ax}\,dx = \frac{1}{a}e^{ax}$

(39) $\int xe^{ax}\,dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax}$

(40) $\int x^2e^{ax}\,dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax}$

(41) $\int \cos ax\,dx = \frac{1}{a}\sin ax$

(42) $\int \sin ax\,dx = -\frac{1}{a}\cos ax$

(43) $\int \tan ax\,dx = -\frac{1}{a}\ln\cos ax$

(44) $\int \frac{1}{\cos x}\,dx = \ln\left|\frac{1}{\cos x} + \tan x\right| = 2\tanh^{-1}\left(\tan\frac{x}{2}\right)$

(45) $\int \frac{1}{\sin x}\,dx = \ln\left|\tan\frac{x}{2}\right|$

(46) $\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$

(47) $\int x^2\cos ax\,dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2 - 2}{a^3}\sin ax$

(48) $\int x\sin ax\,dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2}$

(49) $\int x^2\sin ax\,dx = \frac{2 - a^2x^2}{a^3}\cos ax + \frac{2x\sin ax}{a^2}$

(50) $\int e^{bx}\cos ax\,dx = \frac{1}{a^2+b^2}e^{bx}(a\sin ax + b\cos ax)$

(51) $\int e^{bx}\sin ax\,dx = \frac{1}{a^2+b^2}e^{bx}(b\sin ax - a\cos ax)$

Table of Laplace Transforms

$f(t)$	$\mathcal{L}[f(t)] = F(s)$	
1	$\frac{1}{s}$	(1)
$e^{at}f(t)$	$F(s-a)$	(2)
$\mathcal{U}(t-a)$	$\frac{e^{-as}}{s}$	(3)
$f(t-a)\mathcal{U}(t-a)$	$e^{-as}F(s)$	(4)
$\delta(t)$	1	(5)
$\delta(t-t_0)$	e^{-st_0}	(6)
$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$	(7)
$f'(t)$	$sF(s) - f(0)$	(8)
$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$	(9)
$\int_0^t f(x)g(t-x)dx$	$F(s)G(s)$	(10)
$t^n \ (n = 0, 1, 2, \dots)$	$\frac{n!}{s^{n+1}}$	(11)
$t^x \ (x \geq -1 \in \mathbb{R})$	$\frac{\Gamma(x+1)}{s^{x+1}}$	(12)
$\sin kt$	$\frac{k}{s^2+k^2}$	(13)
$\cos kt$	$\frac{s}{s^2+k^2}$	(14)
e^{at}	$\frac{1}{s-a}$	(15)
$\sinh kt$	$\frac{k}{s^2-k^2}$	(16)
$\cosh kt$	$\frac{s}{s^2-k^2}$	(17)
$\frac{e^{at}-e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}$	(18)
$\frac{ae^{at}-be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}$	(19)
te^{at}	$\frac{1}{(s-a)^2}$	(20)
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$	(21)
$e^{at}\sin kt$	$\frac{k}{(s-a)^2+k^2}$	(22)
$e^{at}\cos kt$	$\frac{s-a}{(s-a)^2+k^2}$	(23)
$e^{at}\sinh kt$	$\frac{k}{(s-a)^2-k^2}$	(24)
$e^{at}\cosh kt$	$\frac{s-a}{(s-a)^2-k^2}$	(25)
$t\sin kt$	$\frac{2ks}{(s^2+k^2)^2}$	(26)
$t\cos kt$	$\frac{s^2-k^2}{(s^2+k^2)^2}$	(27)
$t\sinh kt$	$\frac{2ks}{(s^2-k^2)^2}$	(28)
$t\cosh kt$	$\frac{s^2+k^2}{(s^2-k^2)^2}$	(29)
$\frac{\sin at}{t}$	$\arctan\frac{a}{s}$	(30)
$\frac{1}{\sqrt{\pi t}}e^{-a^2/4t}$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$	(31)
$\frac{a}{2\sqrt{\pi t^3}}e^{-a^2/4t}$	$e^{-a\sqrt{s}}$	(32)
$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s}$	(33)